

Different Electronic Charges in Two-Component Superconductor by Coherente State

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Abstract

Recently, the different electronic charges, which are related to the different coupling constants with magnetic field, in the two-component superconductor have been studied in frame of Ginzburg-Landau theory. In order to study the electronic charges in detail we suggest the wave function in the two-component superconductor to be coherent state. We find the different electronic charges exist not only in the coherent state but the incoherent state. But the ratio of the different charges in the coherent state is different from ratio in the incoherence. The expressions of the coupling constants are given directly based on the coherence effects. We also discuss the winding number in such system.

Keywords: two-component superconductor; Ginzburg-Landau theory; winding number; coherent state

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Introduction

Two-component superconductor [1–4] has inspired great interests since high temperature superconductor MgB_2 was observed in the experiments [5–7]. The reason is that the two-component superconductor exhibits some novel properties. For example, there are composite vortices structure [8–10]; The two-component superconductor is very different from the type I and type II superconductor. In Ref [11, 12], the authors call it type 1.5 superconductor. Recently, the different electronic charges carried by the condensates are studied by using of Ginzburg-Landau theory [13]. They assume that the coupling constants in the two-component superconductor are different. The coupling constants come from the interaction between the superpositions of the condensates and magnetic field. The electronic charges are expressed by the product of the coupling constants and electronic charges carried by Cooper pairs. This is the reason leading to the different electronic charges [13]. The ratio of the two coupling constants is just the ratio of the two different electronic charges, which is rational number. In Ref [13], they find that the different condensates carry the different windings and different fractional vortices by using of the flux quantization and finiteness of the energy per unit length.

The different electric charges can exist in the multicomponent systems, such as two-component superconductor and liquid metallic deuterium. We also suggest the different electric charges exist in the Josephson effect [14]. The reason leading to the different electric charges is the mixtures of condensates. In general, several superpositions are need to describe the condensates in such systems. In order to study the mixtures, it is nature to assume the superpositions to be in coherence, or linear combination. In this letter, we suggest the superpositions of the two condensates in the two-component superconductor are coherent. We devote to study how the coherence effects give rise to the different coupling constants. We also try to find how the coherent state and incoherent state affect the two electronic charges. The ratio of the different charges are studied in the coherent state and incoherent state and we find the ratios are different. Based on ϕ -mapping topological theory [15–17], we discuss which condition leads to integer vorticity and which condition leads to fractional vorticity. The contents are arranged as follows: in section 2, we suggest the superpositions describing the two condensates are coherent. The different electronic charges are studied by using of this coherent state. The expressions of the coupling constants in the coherent state and

incoherent state are deduced directly. In section 3, in the frame of ϕ -mapping topological theory, we find the velocity of the coherent state can carry magnetic flux. The vorticities of the velocity in the coherent state and incoherent state are expressed by winding number. Then the fractional vorticity and magnetic flux are obtained. In section 4, a conclusion is given.

I. DIFFERENT ELECTRONIC CHARGES IN THE TWO-COMPONENT SUPERCONDUCTOR BY COHERENT STATE

The Ginzburg-Landau free energy, which is used to describe the two-component condensate, is expressed as

$$F = \sum_{\alpha} \frac{\hbar^2}{2m} \left[\left(\nabla + i \frac{e}{\hbar c} \mathbf{A} \right) \psi_{\alpha} \right]^2 + V(\psi) + \frac{1}{8\pi} \mathbf{B}^2, \quad \alpha = 1, 2 \quad (1)$$

where $e = 2e^*$ is the electronic charge of the Cooper pair. Here, e^* is the electronic charge. \mathbf{A} is $U(1)$ gauge potential and \mathbf{B} is magnetic field

$$\mathbf{B} = \nabla \times \mathbf{A}. \quad (2)$$

The potential V is

$$V = \sum_{\alpha} \left[a_{\alpha} |\psi_{\alpha}|^2 + \frac{1}{2} b_{\alpha} |\psi_{\alpha}|^4 \right]. \quad (3)$$

where $\alpha = 1, 2$. Let us assume the superpositions ψ_1 and ψ_2 describe two condensates respectively. Then we suggest the total wave function in the two-component superconductor should be coherent state and write it as

$$\Psi = C_1 \psi_1 + C_2 \psi_2. \quad (4)$$

The superposition of every condensate is the anzat $\psi_{\alpha} = \sqrt{\rho_{\alpha}} e^{i\theta_{\alpha}}$. Then the densities of the superpositions respectively are

$$\rho_{\alpha} = \psi_{\alpha} \psi_{\alpha}^* \quad \alpha = 1, 2. \quad (5)$$

By considering the condition of normalization, we have

$$\sum_{\alpha=1}^2 C_{\alpha} C_{\alpha}^* = 1. \quad (6)$$

The density of the coherent state is

$$\rho = \Psi\Psi^* = C_1C_1^*\psi_1\psi_1^* + C_2C_2^*\psi_2\psi_2^* + C_1C_2^*\psi_1\psi_2^* + C_2C_1^*\psi_2\psi_1^*. \quad (7)$$

This density also can be written as

$$\rho = C_1C_1^*\psi_1\psi_1^* + C_2C_2^*\psi_2\psi_2^* + 2\operatorname{Re}[C_1C_2^*\psi_1\psi_2^*]. \quad (8)$$

When the wave function Ψ is in coherent state, the term in (8) $\operatorname{Re}[C_1C_2^*\psi_1\psi_2^*] \neq 0$, which exhibits coherent effect. When the wave function Ψ is in incoherent state, the term $\operatorname{Re}[C_1C_2^*\psi_1\psi_2^*] = 0$, the coherent effect of the system disappears. Support the Schrödinger equation of a charged particle without spin in a electromagnetic field is

$$i\hbar\frac{\partial\psi}{\partial t} = H\psi, \quad (9)$$

where $H = -\frac{\hbar^2}{2m}(\nabla + i\frac{e}{\hbar c}\mathbf{A})^2 + V$. Let ψ_1 and ψ_2 be the solutions of the Schrödinger equation, then the coherent state Ψ is also the solution of the Schrödinger equation. It should be noted that the potential V is real number and it has no contribution to probability current. Then the probability current of the coherent state satisfies

$$\delta H = -\frac{1}{c} \int \mathbf{J} \cdot \delta \mathbf{A} dV. \quad (10)$$

Then the probability current is

$$\mathbf{J} = \frac{i\hbar}{2m} \left(\Psi^* \left(\nabla + i\frac{e}{\hbar c}\mathbf{A} \right) \Psi - \Psi \left(\nabla + i\frac{e}{\hbar c}\mathbf{A} \right) \Psi^* \right), \quad (11)$$

which is just the superconducting current. Put (4) into this formula, we have

$$\begin{aligned} \mathbf{J} = & \frac{i\hbar}{2m} \sum_{\alpha=1}^2 C_{\alpha}C_{\alpha}^* (\psi_{\alpha}^* \nabla \psi_{\alpha} - \psi_{\alpha} \nabla \psi_{\alpha}^*) \\ & + \frac{i\hbar}{2m} C_1C_2^* (\psi_2^* \nabla \psi_1 - \psi_1 \nabla \psi_2^*) + \frac{i\hbar}{2m} C_1C_2^* (\psi_1^* \nabla \psi_2 - \psi_2 \nabla \psi_1^*) - \frac{e}{mc} \mathbf{A} \Psi \Psi^*. \end{aligned} \quad (12)$$

Substitute the electronic charge into this formula, the current will become to the electronic current. We write it as

$$\begin{aligned} \mathbf{J}_e = & \frac{i\hbar e}{2m} \sum_{\alpha=1}^2 C_{\alpha}C_{\alpha}^* (\psi_{\alpha}^* \nabla \psi_{\alpha} - \psi_{\alpha} \nabla \psi_{\alpha}^*) \\ & + \frac{i\hbar e}{2m} C_1C_2^* (\psi_2^* \nabla \psi_1 - \psi_1 \nabla \psi_2^*) + \frac{i\hbar e}{2m} C_1C_2^* (\psi_1^* \nabla \psi_2 - \psi_2 \nabla \psi_1^*) - \frac{e^2}{mc} \mathbf{A} \Psi \Psi^*. \end{aligned} \quad (13)$$

The second and third terms show the coherence effects of the two superpositions and we denote these terms as \mathbf{J}_{e2} . The other terms are denoted as \mathbf{J}_{e1} .

$$\mathbf{J}_{e1} = \frac{i\hbar e}{2m} \sum_{\alpha=1}^2 C_{\alpha} C_{\alpha}^* (\psi_{\alpha}^* \nabla \psi_{\alpha} - \psi_{\alpha} \nabla \psi_{\alpha}^*) - \frac{e^2}{mc} \mathbf{A} \Psi \Psi^*, \quad (14)$$

$$\mathbf{J}_{e2} = \frac{i\hbar e}{2m} C_1 C_2^* (\psi_2^* \nabla \psi_1 - \psi_1 \nabla \psi_2^*) + \frac{i\hbar e}{2m} C_1 C_2^* (\psi_1^* \nabla \psi_2 - \psi_2 \nabla \psi_1^*). \quad (15)$$

Let us define new complex variable as

$$\Lambda = C_1 C_2^* \psi_1 \psi_2^*, \quad (16)$$

then the complex conjugation of this variable is

$$\Lambda^* = C_1^* C_2 \psi_1^* \psi_2. \quad (17)$$

These complex numbers also can be rewritten as

$$\begin{aligned} \Lambda &= \Lambda_1 + i\Lambda_2, \\ \Lambda^* &= \Lambda_1 - i\Lambda_2, \end{aligned} \quad (18)$$

where Λ_1 and Λ_2 are real numbers. Put the formulas (16,17,18) into the density (8), we get the density of the wave function

$$\rho = |C_1|^2 \rho_1 + |C_2|^2 \rho_2 + 2\Lambda_1. \quad (19)$$

These parameters provide us one way to determine the coherence effects of the system. When $\Lambda = 0$, the wave function in the two-component superconductor is the incoherent state. When $\Lambda \neq 0$, the wave function is the coherent state. From (15), the electronic current \mathbf{J}_{e2} is deduced as

$$\mathbf{J}_{e2} = \frac{i\hbar e}{2m} \Lambda_1 \left[\frac{(\psi_1^* \nabla \psi_1 - \psi_1 \nabla \psi_1^*)}{\psi_1 \psi_1^*} + \frac{(\psi_2^* \nabla \psi_2 - \psi_2 \nabla \psi_2^*)}{\psi_2 \psi_2^*} \right] - \frac{\hbar e \Lambda_2}{2m} \nabla \ln \left(\frac{\psi_1 \psi_1^*}{\psi_2 \psi_2^*} \right). \quad (20)$$

The electronic current \mathbf{J}_{e1} can be given as

$$\mathbf{J}_{e1} = \frac{i\hbar e}{2m} \sum_{\alpha} |C_{\alpha}|^2 \rho_{\alpha} \left(\frac{\psi_{\alpha}^* \nabla \psi_{\alpha} - \psi_{\alpha} \nabla \psi_{\alpha}^*}{\psi_{\alpha} \psi_{\alpha}^*} \right) - \frac{e^2}{mc} \mathbf{A} \Psi \Psi^*. \quad (21)$$

Finally, the total electronic current is

$$\begin{aligned} \mathbf{J}_e &= \frac{i\hbar e}{2m} (|C_1|^2 \rho_1 + \Lambda_1) \frac{(\psi_1^* \nabla \psi_1 - \psi_1 \nabla \psi_1^*)}{\psi_1 \psi_1^*} + \frac{i\hbar e}{2m} (|C_2|^2 \rho_2 + \Lambda_1) \frac{(\psi_2^* \nabla \psi_2 - \psi_2 \nabla \psi_2^*)}{\psi_2 \psi_2^*} \\ &\quad - \frac{\hbar e \Lambda_2}{2m} \nabla \ln \left(\frac{\rho_1}{\rho_2} \right) - \frac{e^2}{mc} \mathbf{A} \Psi \Psi^*. \end{aligned} \quad (22)$$

It should be noted that the term $\nabla \ln \left(\frac{\rho_1}{\rho_2} \right)$ is vector and $\ln \left(\frac{\rho_1}{\rho_2} \right)$ can be denoted as θ . By using of these definitions, the vector is

$$\nabla \theta(x) = \nabla \ln \left(\frac{\rho_1}{\rho_2} \right). \quad (23)$$

When the wave function in the two-component superconductor is the incoherent state, $\Lambda_1 = \Lambda_2 = 0$, the term $\nabla \theta(x)$ disappears. The electronic current is same as the current obtained by $\frac{\delta F}{\delta A}$. When the wave function is the coherent state, $\Lambda_1 \neq 0, \Lambda_2 \neq 0$, the term $\nabla \theta(x)$ is analyzed as follows: if $\frac{\rho_1}{\rho_2}$ is a function of coordinates and the function $\theta(x)$ satisfies Clairaut's theorem, the second mixed derivatives of $\theta(x)$ is symmetry

$$\partial_i \partial_j \theta(x) = \partial_j \partial_i \theta(x). \quad (24)$$

This shows

$$\nabla \times \nabla \theta(x) = 0. \quad (25)$$

Then the function $\theta(x)$ can be seen as the phase of the $U(1)$ gauge magnetic potential, the $U(1)$ gauge potential is translated as

$$\mathbf{A}' \rightarrow \mathbf{A} + \nabla \theta. \quad (26)$$

Because of Eq.(25), the magnetic field under this translation is $\mathbf{B}' = \nabla \times \mathbf{A}' = \mathbf{B}$. The phase has no contribution to the magnetic field. This result shows that the $U(1)$ -like phase θ does not lead to observed phenomenon. It is natural to define the currents as \mathbf{j}_1 and \mathbf{j}_2 respectively

$$\begin{aligned} \mathbf{j}_1 &= \rho_1 \mathbf{V}_1, \\ \mathbf{j}_2 &= \rho_2 \mathbf{V}_2, \end{aligned} \quad (27)$$

where

$$\begin{aligned} \mathbf{V}_1 &= \frac{(\psi_1^* \nabla \psi_1 - \psi_1 \nabla \psi_1^*)}{\psi_1 \psi_1^*}, \\ \mathbf{V}_2 &= \frac{(\psi_2^* \nabla \psi_2 - \psi_2 \nabla \psi_2^*)}{\psi_2 \psi_2^*}. \end{aligned} \quad (28)$$

Then the total electronic current is

$$\mathbf{J}_e = \frac{i\hbar e}{2m} (|C_1|^2 \rho_1 + \Lambda_1) \mathbf{V}_1 + \frac{i\hbar e}{2m} (|C_2|^2 \rho_2 + \Lambda_1) \mathbf{V}_2 - \frac{e^2}{mc} \mathbf{A} \Psi \Psi^* - \frac{\hbar e \Lambda_2}{2m} \nabla \theta(x). \quad (29)$$

In order to study the electronic charges in the two-component superconductor, we write the electronic current as

$$\mathbf{J}_e = \frac{i\hbar e}{2m} \left(|C_1|^2 + \frac{\Lambda_1}{\rho_1} \right) \mathbf{j}_1 + \frac{i\hbar e}{2m} \left(|C_2|^2 + \frac{\Lambda_1}{\rho_2} \right) \mathbf{j}_2 - \frac{e^2}{mc} \mathbf{A} \Psi \Psi^* - \frac{\hbar e \Lambda_2}{2m} \nabla \theta(x), \quad (30)$$

here, we use the relations (27). When the wave function is the coherent state, $\Lambda_1 \neq 0$, then the electronic charges in the superconductor are

$$\begin{aligned} e_1 &= \left(|C_1|^2 + \frac{\Lambda_1}{\rho_1} \right) e, \\ e_2 &= \left(|C_2|^2 + \frac{\Lambda_1}{\rho_2} \right) e. \end{aligned} \quad (31)$$

The ratio of the two charges is

$$\frac{e_1}{e_2} = \frac{\left(|C_1|^2 + \frac{\Lambda_1}{\rho_1} \right)}{\left(|C_2|^2 + \frac{\Lambda_1}{\rho_2} \right)}. \quad (32)$$

Where we find the coupling constants g_1 and g_2 in Ref. [13] must be

$$\begin{aligned} g_1 &= |C_1|^2 + \frac{\Lambda_1}{\rho_1}, \\ g_2 &= |C_2|^2 + \frac{\Lambda_1}{\rho_2}. \end{aligned} \quad (33)$$

When the wave function is the incoherent state, $\Lambda_1 = \Lambda_2 = 0$, then the electronic current is

$$\mathbf{J}_e = \frac{i\hbar |C_1|^2 e}{2m} \mathbf{j}_1 + \frac{i\hbar |C_2|^2 e}{2m} \mathbf{j}_2 - \frac{e^2}{mc} \mathbf{A} \Psi \Psi^*.$$

In this case the electronic charges are $e_1 = |C_1|^2 e$ and $e_2 = |C_2|^2 e$. Therefore, we find g_1 and g_2 must be

$$\begin{aligned} g_1 &= |C_1|^2 \\ g_2 &= |C_2|^2. \end{aligned} \quad (34)$$

The ratio of the two charges is

$$\frac{e_1}{e_2} = \frac{|C_1|^2}{|C_2|^2}. \quad (35)$$

Considering $\mathbf{J}_e = \rho \mathbf{V}$, the velocity of the wave function is

$$\mathbf{V} = \frac{i\hbar e}{2m} \frac{(|C_1|^2 \rho_1 + \Lambda_1)}{\rho} \mathbf{V}_1 + \frac{i\hbar e}{2m} \frac{(|C_2|^2 \rho_2 + \Lambda_1)}{\rho} \mathbf{V}_2 - \frac{e^2}{mc} \mathbf{A} - \frac{\hbar e \Lambda_2}{2m\rho} \nabla \theta(x). \quad (36)$$

All these show that the different electronic charges in the two-component superconductor exist not only in the coherent state but the incoherent state. But from (32) and (35), the ratio of the difference electronic charges in the coherence is different from the the ratio in the incoherence. The same analyze can be done for the coupling constants. Although the interaction between two condensates are mediated by the $U(1)$ magnetic field, the reason leading to different electronic charges, or coupling constants, is that there are coherence effects in the two-component superpositions. It is interesting that when the wave function is in the coherence, there is a $U(1)$ -like phase adding to the gauge potential \mathbf{A} . This phase has no contribution to magnetic field. When the wave function is the incoherent state, the phase disappears and has no contribution to magnetic field too. So the coherence effects do not affect the magnetic field.

II. WINDING NUMBER IN THE TWO-COMPONENT SUPERCONDUCTOR

Topology has play an important role in mathematics and physics. Topological defect, or phase singularity, appears in many physics system. Based on the ϕ -mapping topological theory, the direct and explicit expression of the isolated singular topological structures in the continuous media have been mathematically deduced, such as monopoles [15], superconductor [10], Chern-Simons vortex [18]. In ϕ -mapping topological theory, the wave function $\phi(x^i)$ ($i = 1, 2, 3$) is 2-dimensional vector field can be looked upon as a smooth map $\phi : X \rightarrow R^2$. X is a 3-dimensional Riemannian manifold and R^2 is a 2-dimensional Euclidean space. When $\phi = 0$ at point p , the topological properties of this map are non-trivial. The zero point p is called a singular point of ϕ . By using of ϕ -mapping topological theory, a topological number, called winding number, is obtained. It can be used to describe the topological properties of singular point. The superpositions of the two condensates are expressed as

$$\begin{aligned}\psi_1 &= \psi_1^1 + i\psi_1^2, \\ \psi_2 &= \psi_2^1 + i\psi_2^2.\end{aligned}\tag{37}$$

By using of ϕ -mapping topological theory, the unit vectors n^a must be introduced.

$$\begin{aligned} n_1^a &= \frac{\psi_1^a}{\|\psi_1\|}, \\ n_2^a &= \frac{\psi_2^a}{\|\psi_2\|} \quad a = 1, 2. \end{aligned} \quad (38)$$

It is found that the unit vectors exhibit some singularities at where the wave function is zero. At the zero point, the phase of the wave function is uncertain. So the nontrivial topological properties can be deduced from the unit vectors directly [15, 17]. The unit vectors should satisfy $n_{1(2)}^a n_{1(2)}^a = 1$. The velocities of the two superpositions are calculated from these unit vectors as

$$\begin{aligned} V_{1i} &= \frac{\hbar}{m} \epsilon_{ab} n_1^a \partial_i n_1^b, \\ V_{2i} &= \frac{\hbar}{m} \epsilon_{ab} n_2^a \partial_i n_2^b. \end{aligned} \quad (39)$$

The curl of the velocity is

$$\begin{aligned} \nabla \times \mathbf{V} &= \frac{i\hbar e}{2m} \frac{(|C_1|^2 \rho_1 + \Lambda_1)}{\rho} \nabla \times \mathbf{V}_1 + \frac{i\hbar e}{2m} \frac{(|C_2|^2 \rho_2 + \Lambda_1)}{\rho} \nabla \times \mathbf{V}_2 \\ &\quad - \frac{e^2}{mc} \nabla \times \mathbf{A} - \frac{\hbar e \Lambda_2}{2m\rho} \nabla \times \nabla \theta(x). \end{aligned} \quad (40)$$

By considering (25), the curl of the velocity is

$$(\nabla \times \mathbf{V})_i = \frac{\hbar e}{m} \frac{(|C_1|^2 \rho_1 + \Lambda_1)}{\rho} \epsilon^{ijk} \epsilon_{ab} \partial_j n_1^a \partial_k n_1^b + \frac{\hbar e}{m} \frac{(|C_2|^2 \rho_2 + \Lambda_1)}{\rho} \epsilon^{ijk} \epsilon_{ab} \partial_j n_2^a \partial_k n_2^b - \frac{e^2}{mc} (\nabla \times \mathbf{A})_i. \quad (41)$$

We find that there is magnetic flux carried by the velocity through rewriting the curl of the velocity as

$$(\nabla \times \mathbf{V})_i = \frac{e^2}{mc} \left[\frac{\hbar c}{e} \frac{(|C_1|^2 \rho_1 + \Lambda_1)}{\rho} \epsilon^{ijk} \epsilon_{ab} \partial_j n_1^a \partial_k n_1^b + \frac{\hbar c}{e} \frac{(|C_2|^2 \rho_2 + \Lambda_1)}{\rho} \epsilon^{ijk} \epsilon_{ab} \partial_j n_2^a \partial_k n_2^b - (\nabla \times \mathbf{A})_i \right]. \quad (42)$$

The vorticity of the velocity (36) will be

$$\begin{aligned} \int (\nabla \times \mathbf{V}) \cdot d\mathbf{S} &= \frac{e^2}{mc} \left[\frac{\hbar c}{e} \frac{(|C_1|^2 \rho_1 + \Lambda_1)}{\rho} \int \epsilon^{ijk} \epsilon_{ab} \partial_j n_1^a \partial_k n_1^b \mathbf{e}_i \cdot d\mathbf{S} \right] \\ &\quad + \frac{e^2}{mc} \left[\frac{\hbar c}{e} \frac{(|C_2|^2 \rho_2 + \Lambda_1)}{\rho} \int \epsilon^{ijk} \epsilon_{ab} \partial_j n_2^a \partial_k n_2^b \mathbf{e}_i \cdot d\mathbf{S} \right] + \frac{e^2}{mc} \int (\nabla \times \mathbf{A}) \cdot d\mathbf{S}. \end{aligned} \quad (43)$$

We note that the integral $\int (\nabla \times \mathbf{A}) \cdot d\mathbf{S} = \int \mathbf{B} \cdot d\mathbf{S}$ in the last term is magnetic flux. The magnetic flux quanta is $\Phi_0 = \frac{hc}{e}$, then the magnetic flux can be expressed as

$$\Phi = W'' \Phi_0 = W'' \frac{hc}{e}. \quad (44)$$

According to ϕ -mapping topological theory, the first term and second term will be

$$\begin{aligned} \int (\nabla \times \mathbf{V}_1) \cdot d\mathbf{S} &= \int \epsilon^{ijk} \epsilon_{ab} \partial_j n_1^a \partial_k n_1^b \mathbf{e}_i \cdot d\mathbf{S} = 2\pi W_1, \\ \int (\nabla \times \mathbf{V}_2) \cdot d\mathbf{S} &= \int \epsilon^{ijk} \epsilon_{ab} \partial_j n_2^a \partial_k n_2^b \mathbf{e}_i \cdot d\mathbf{S} = 2\pi W_2, \end{aligned} \quad (45)$$

where W is called the winding number. The meaning of the winding number is: let us consider that the wave function ψ is the map: $R^3 \rightarrow C^1$, when the coordinates x travel around the zero point of the wave function once, the wave function covers the zero point W times. In general, the winding number is integer number. The third term is the magnetic flux and it is quanta. Finally, the vorticity of the total velocity is given as

$$\int (\nabla \times \mathbf{V}) \cdot d\mathbf{S} = \frac{e^2}{mc} \left[\frac{(|C_1|^2 \rho_1 + \Lambda_1)}{\rho} W_1 \Phi_0 + \frac{(|C_2|^2 \rho_2 + \Lambda_1)}{\rho} W_2 \Phi_0 + W'' \Phi_0 \right]. \quad (46)$$

This formula shows the velocity of the coherent state carries magnetic flux, it is

$$\Phi = \frac{(|C_1|^2 \rho_1 + \Lambda_1)}{\rho} W_1 \Phi_0 + \frac{(|C_2|^2 \rho_2 + \Lambda_1)}{\rho} W_2 \Phi_0 + W'' \Phi_0. \quad (47)$$

The magnetic flux may be integer or fraction. By considering formula (45), the vorticity of the total velocity should satisfy the relation (45), we have

$$\int (\nabla \times \mathbf{V}) \cdot d\mathbf{S} = \frac{he}{m} W'. \quad (48)$$

The magnetic flux from (47) is

$$W' \Phi_0 = \frac{(|C_1|^2 \rho_1 + \Lambda_1)}{\rho} W_1 \Phi_0 + \frac{(|C_2|^2 \rho_2 + \Lambda_1)}{\rho} W_2 \Phi_0 + W'' \Phi_0. \quad (49)$$

Then the winding number should satisfy the relation

$$W' = \frac{(|C_1|^2 \rho_1 + \Lambda_1)}{\rho} W_1 + \frac{(|C_2|^2 \rho_2 + \Lambda_1)}{\rho} W_2 + W''. \quad (50)$$

For simple, we consider the wave function is the incoherent state, that is $\Lambda_1 = 0$. The relation (50) translates to

$$W' - W'' = \frac{|C_1|^2 \rho_1}{\rho} W_1 + \frac{|C_2|^2 \rho_2}{\rho} W_2. \quad (51)$$

Let us denote $W = W' - W''$, (51) changes to

$$W = \frac{|C_1|^2 \rho_1}{\rho} W_1 + \frac{|C_2|^2 \rho_2}{\rho} W_2. \quad (52)$$

Based on the formula $\rho = |C_1|^2 \rho_1 + |C_2|^2 \rho_2$, so we can define

$$\begin{aligned} \frac{q}{p} &= \frac{|C_1|^2 \rho_1}{\rho}, \\ \frac{p-q}{p} &= \frac{|C_2|^2 \rho_2}{\rho}. \end{aligned} \quad (53)$$

where p, q are integer number and $q < p$. Then the total vorticity is

$$W = \frac{q}{p} W_1 + \left(\frac{p-q}{p} \right) W_2. \quad (54)$$

When the winding numbers $W_1 = W_2$, we find the total winding number is W , which is integer. By considering (47), the magnetic flux is integer. When $W_1 = l_1 p$ and $W_2 = l_2 p$, l_1, l_2 are integer number, the total winding number is $W = q l_1 + (p-q) l_2$. The winding number and magnetic flux are integer. When W_1 and W_2 are other numbers, the total winding number and magnetic flux are fractional. When the wave function is the coherent state, let us write

$$\frac{q'}{p+2q'} = \frac{\Lambda_1}{\rho}, \quad \frac{q+q'}{p+2q'} = \frac{|C_1|^2 \rho_1 + \Lambda_1}{\rho}, \quad \frac{p-q+q'}{p+2q'} = \frac{|C_2|^2 \rho_2 + \Lambda_1}{\rho}. \quad (55)$$

Then the total winding number is

$$W = \frac{q+q'}{p+2q'} W_1 + \left(\frac{p-q+q'}{p+2q'} \right) W_2. \quad (56)$$

When $W_1 = W_2$, the total winding number and magnetic flux are integer. Let $W_1 = l_1 (p+2q')$ and $W_2 = l_2 (p+2q')$, the total winding number is $W = (q+q') l_1 + (p-q+q') l_2$. This winding number and magnetic flux are integer. When W_1 and W_2 are other number, the total winding number and magnetic flux are fractional.

III. CONCLUSION

In this letter, we suggest that the wave function in the two-component superconductor is a coherent state $\Psi = C_1 \psi_1 + C_2 \psi_2$. Where ψ_1 and ψ_2 are the superpositions of the two condensates respectively. Then we deduce the probability current and electronic current of

the coherent state based on Ginzburg-Landau free energy and Schrödinger equation. When the wave function is the coherent state, the coupling constants are $g_1 = |C_1|^2 + \frac{\Lambda_1}{\rho_1}$ and $g_2 = |C_2|^2 + \frac{\Lambda_1}{\rho_2}$. At same time, there is vector $\nabla\theta$, which can be seen as the phase of $U(1)$ magnetic potential, arising from the coherence effects. This phase has no contribution to the magnetic field. When the wave function is the incoherent state, the coupling constants are $g_1 = |C_1|^2$ and $g_2 = |C_2|^2$. From these, the ratio of the two coupling constants in the coherence is different from the ratio in the incoherence. Then we find the reason leading to different electronic charges is the coherence effects although there is $U(1)$ magnetic field.

The coherence effects are not only the reason leading to the different electronic charges but also fractional vorticity and magnetic flux. In frame of ϕ -mapping topological theory, we find that the vorticity can be expressed by winding number. The vorticity of the wave function can carry magnetic flux. When the total winding number is integer, the magnetic flux is integer. When the total winding number is fractional, the magnetic flux is fractional.

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